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Discussion Papers in
Economics and Econometrics

Title: Inflation versus price-level targeting with short and long-term nominal debt

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No. 1505

ISSN 0966-4246

Inflation versus price-level targeting with short and long-term nominal debt

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31 July, 2015

Abstract

This paper presents a DSGE model in which the government issues short and long-term nominal debt. The model is used to compare social welfare under inflation targeting (IT) and price-level targeting (PT) monetary regimes. When the share of long-term debt is calibrated to be positive as in the data, PT raises social welfare relative to IT because it lowers long-term inflation risk. However, if the share of long-term debt is set optimally to maximise social welfare, the welfare gains of PT are eliminated because only short-term debt should be issued. These results are robust to calibration, but the presence of productivity risk is crucial.

Keywords: government debt; monetary policy; inflation targeting, price-level targeting.
JEL classification: E52, E63

1 Introduction

When inflation is unanticipated, real returns on nominal assets vary, implying fluctuations in consumption and wealth. An important channel through which this happens is the change in the real value of nominal government debt held by the private sector. The maturity of bond portfolios is an important part of this equation because short and long-term nominal claims need not be revalued equally in the face of inflationary shocks. This observation motivates a comparison of inflation targeting (IT) and price-level targeting (PT) regimes. Under IT, unanticipated shocks to inflation are not reversed by policy, so an unanticipated rise in inflation erodes both short-term and long-term nominal claims. By contrast, PT offsets unanticipated shocks to inflation, so unanticipated inflation erodes short-term nominal claims but leaves purchasing power of long-term nominal claims largely unaffected.[2]

In this paper, a DSGE model with short and long-term nominal debt is used to compare social welfare under IT and PT regimes. In recent years, both policymakers and academics have been interested in this comparison. Several papers have shown that PT offers short-term stabilisation benefits over IT when agents are forward-looking. For instance, Vestin (2006) shows that in the baseline New Keynesian model, PT reduces inflation variability for a given level of output gap variability if policymakers operate under discretion. In the same model, optimal commitment implies a stationary price level (Clarida et al., 1999) and interest rate rules which respond to the price level outperform standard Taylor rules both with and without the zero lower bound on nominal interest rates (Eggertsson and Woodford, 2003; Nakov, 2008; Giannoni, 2014). In light of these results, the Bank of Canada recently conducted an

[1] Email: M.C.Hatcher@soton.ac.uk. I am grateful to Alex Mennuni and Chiara Forlati for helpful comments. This project was started while I received funding from ESRC Postdoctoral Fellowship PTA-026-27-2964.
[2] In econometric jargon, the price level follows a random walk under IT but is trend-stationary under PT.

assessment of the costs and benefits of PT (see Bank of Canada, 2011).[3] However, to the author's knowledge, no paper has compared IT and PT using a model with short and long-term government debt in which long-term inflation risk matters for social welfare.[4] Since government debt accounts for a non-trivial fraction of net nominal wealth in developed economies (Doepke and Schneider, 2006; Meh and Terajima, 2011), this analysis may be important for making a full assessment of the relative merits of PT.

An overlapping generations (OG) model in the spirit of Samuelson (1958) and Diamond (1965) is set out. The model has three features that make it useful for assessing the interaction between the share of long-term debt and social welfare under IT and PT. First, as noted by Barro (1974), government debt is net wealth in standard OG models. As a result, there is scope for debt policy to have meaningful effects on social welfare.[5] Second, the OG model captures the implications of unanticipated inflation variations for the real value of government debt held by the private sector. In the model, there are two channels through which inflation risk affects social welfare: (i) fluctuations in the real returns on government debt lead to variations in asset income and taxes, and (ii) short and long-term inflation risk affect the inflation risk premium the government pays on short and long-term debt and hence the average level of taxes needed to satisfy the government budget constraint.[6] Third, OG models provide a tractable framework for modelling inflation risk and asset prices over a long horizon without introducing a large number of state variables. This allows the analysis to proceed in a simple model that emphasises intuition but captures the key policy trade-offs.

The main findings are as follows. In an economy where the share of long-term debt is calibrated to be positive as in the data, PT implies welfare gains relative to IT because it raises consumption by the young and middle-aged and lowers consumption risk at these ages. The benefits of PT are related to the fact that it lowers long-term inflation risk relative to IT. First, because PT lowers long-term inflation risk, it lowers the inflation risk premium on long-term debt, which means that taxes are lower on average under PT because it is less expensive for the government to borrow using long-term debt.[7] As a result, average consumption by the young and middle-aged (who are taxed) is higher under a PT regime. Second, because long-term inflation risk is lower under a PT regime, the real return on long-term debt is less variable than under IT. As a result, the young and middle-aged face less variable taxes than under IT, so that their consumption risk is lower. However, if the share of long-term debt is set optimally under both IT and PT, the above conclusions change dramatically. In particular, since it is optimal to issue only short-term debt under both regimes, there is nothing to choose between IT and PT in terms of consumption levels, consumption risk or social welfare. This result casts doubt on the conventional wisdom that PT would raise social welfare in a world with long-term nominal contracts because it reduces long-term inflation risk.

[3] For surveys of the recent literature on price-level targeting, see Ambler (2009), Crawford et al. (2009), Bank of Canada (2011) and Hatcher and Minford (forthcoming). Dittmar et al. (1999) and Gavin et al. (2009) examine the implications of IT and PT for long-term inflation risk in theoretical models.
[4] Indeed, formal studies of the effects of PT outside business cycle frequencies are rare in the DSGE literature. Exceptions include Meh et al. (2010) and Hatcher (2014).
[5] Minford and Peel (2002, Ch. 12) provide a neat illustration of this result.
[6] In linear or log-linearized models there is 'certainty equivalence', so that risk premia are zero. Kim and Kim (2003) show that failure to account for the effects of risk can lead to spurious welfare reversals. We therefore consider a second-order approximation of the model where risk-premia are non-zero (but constant).
[7] See Bekaert and Wang (2010) for a survey of the inflation risk premium. Here, it is defined as the difference between the expected real return on nominal debt and the risk-free real interest rate (see Appendix A1).

The main results of the paper are robust in several directions. For instance, the baseline results are robust to alternative parameterizations of the model and the treatment of the utility of initial generations. However, the result that issuing only short-term debt is optimal under both regimes hinges on the presence of productivity risk. With productivity risk, it is optimal to issue only short-term debt even if inflation risk is absent. But if productivity risk is absent, it is optimal to issue only short-term debt under IT, but not PT. As a result, there are welfare gains to PT in this case even if the share of long-term is set optimally. It is important to note, however, that this result disappears if even a small amount of productivity risk is introduced.

The paper is related to two main strands of literature. The first is on the implications of unanticipated inflation under IT and PT. In a seminal paper, Doepke and Schneider (2006) document postwar nominal portfolios in the US and show that unanticipated inflation had substantial redistribution through revaluations of nominal assets and liabilities. Meh and Terajima (2011) later documented nominal portfolios in Canada. Building on these two papers, Meh et al. (2010) simulated aggregate and welfare effects from *one-off* episodes of unanticipated inflation in Canada under IT and PT in a quantitative OG model. They find that there are larger redistributions under IT because long-term nominal contracts undergo larger revaluations in presence base-level drift as unanticipated shocks to the price-level are not offset. Consequently, induced welfare effects are somewhat larger under IT than PT. However, nominal portfolios are assumed to remain fixed across IT and PT in this analysis and the focus is on *one-off* episodes of unanticipated inflation and not *inflation risk* as here.[8]

The second strand of literature to which the paper is related is on optimal nominal contracting under IT and PT. The seminal paper in this literature is Minford et al. (2003), which builds on the insights of Gray (1976) about optimal wage indexation in the face of real and nominal shocks. Minford et al. (2003) build a general equilibrium model in which households have an incentive to smooth real wage fluctuations but cannot access financial markets. They show that optimal wage indexation is substantially lower under a PT regime, where money supply shocks are temporary, than under IT regime where they are permanent. Subsequently, Amano et al. (2007) showed that optimal indexation remains lower under a PT regime even if agents have unrestricted access to financial markets. The study of optimal indexation of government debt was taken up by Hatcher (2014). He showed that if indexed government debt is linked to the price level with a lag, issuing only indexed debt is optimal under IT but not under PT. This paper uses a similar framework, but the key difference in the analysis here is that the maturity of government debt contracts is allowed to vary across regimes.[9]

The paper proceeds as follows. Section 2 presents the model. Section 3 discusses the conduct of monetary policy under IT and PT. Section 4 discusses the optimal policy problem of the government and its solution. In Section 5 the model is calibrated. Section 6 reports the baseline results. Section 7 investigates robustness. Finally, Section 8 concludes.

[8] That is, they consider individual draws from the distribution of inflation shocks and not the entire distribution. As the authors note, the latter is necessary to account for the impact of higher-order moments on social welfare.
[9] In practice, governments issue both short and long-term nominal debt. For example, in 2010, 30-year nominal debt was around 20% of marketable debt outstanding in Canada (Department of Finance Canada 2011, Chart 2) and nominal debt with maturities exceeding 15 years was around 25% of the UK gilt portfolio (DMO, 2010).

2. Model

An overlapping generations (OG) model with three-period lifetimes is considered. The model contains three sectors: a household sector, a government sector, and a productive sector devoted to the production of a single output good. Each of these sectors is described below. This section also discusses the equilibrium conditions of the model and social welfare.

2.1 Consumers

Each generation lives for three periods. The subscripts $\{y, m, o\}$ denote youth, middle-age, and old age. Each period lasts 20 years and the number of generations born per period is constant and normalized to 1. The young and middle-aged supply labour inelastically to firms (with total labour supply per lifetime normalized to 1) and use their wage income to consume and to invest in an optimal portfolio of assets. The old are retired and there is no bequest motive, so they consume all their wealth. The menu of assets available to households includes nominal government debt, physical capital, k, and money, m. The young may invest in either long-term debt b^l or short-term debt $b^{s,y}$, while the middle-aged can invest in short-term debt, denoted $b^{s,m}$.[10] Accordingly, the total amount of debt in the economy in any period is $b^{tot} = b^l + b^{s,y} + b^{s,m}$. Both the young and middle-aged can invest in capital and money, so total capital and real money balances are given by $k = k_y + k_m$ and $M/P = m_y + m_m$, where P is the aggregate price level. The wage income of the young and middle-aged is taxed at rate τ.[11]

Government debt is nominal and default-free. Short-term debt matures in *one period* and pays a nominal interest rate R^s on maturity. Consequently, the real return on short-term debt is R^s/Π, where Π is the one-period gross inflation rate. Long-term debt matures in *two periods* and pays a nominal interest rate R^l on maturity. The real return on long-term debt is given by R^l/Π^l, where Π^l is the two-period gross inflation rate. The nominal interest rates R^s and R^l are endogenously determined and ensure that, for both types of debt, demand is equal to supply. Capital pays a risky real return r^k, which is equal to its marginal product. There is no secondary market for government debt, so long-term bonds can only be purchased in the period in which they are issued and held until maturity, as in Sargent (1987, p. 102-105), Andrés et al. (2004), and Ellison and Tischbirek (2014), amongst others.[12]

Money pays a real return of $r^m = 1/\Pi$. A positive demand for money arises from a reserve requirement as in Champ and Freeman (1990) that requires the young and middle-aged to hold real money balances of at least $\delta > 0$, so that $m_y \geq \delta$ and $m_m \geq \delta$. The main advantage of this assumption is that money has value without having to offer any transactions services, so that any difference in the results under IT and PT can be attributed to the implications of these regimes for long-term inflation risk. The reserve requirement holds with equality provided that $R^s > 1$,[13] which was met in all numerical simulations in this paper.

We therefore have that

$$m_{t,y} = m_{t,m} = \delta, \qquad \forall t \tag{1}$$

[10] The middle-aged would not choose to hold long-term debt since it matures after their final period of life.
[11] This tax is not distortionary because labour supply is inelastic. Levying a lump-sum tax would have the same impact but is less convenient from a calibration perspective because it has no obvious counterpart in the data.
[12] It is worth noting that empirical work suggests many investors follow ‘buy and hold’ strategies, perhaps due to transaction costs of participating in secondary markets. For instance, Ameriks and Zeldes (2004, p. 31) found that “the vast majority of households make few or no changes over time to their portfolio allocations.”
[13] This condition is derived in Section 1.B of the Supplementary Appendix.

Total demand for real money balances is given by $M_t / P_t = m_{t,y} + m_{t,m} = 2\delta$. The stock of fiat money M_t is increased according to simple rules that implement IT and PT, as shown in Section 3. New money is injected into the economy via lump-sum monetary transfers T^1 and T^2 to the middle-aged and old in proportion to their money holdings. Thus, the current money injection is split equally between transfers to these two generations:

$$T_t^1 = T_t^2 = \frac{1}{2}(M_t - M_{t-1}) \tag{2}$$

The budget constraints faced by the generation born in period t are as follows:

$$c_{t,y} = w_t(1-\tau_t)L_y - b_{t+1}^l - b_{t+1}^{s,y} - k_{t+1,y} - m_{t,y} \tag{3}$$

$$c_{t+1,m} = w_{t+1}(1-\tau_{t+1})L_m + r_{t+1}^k k_{t+1,y} + (R_t^s / \Pi_{t+1})b_{t+1}^{s,y} + T_{t+1}^1 / P_{t+1} + r_{t+1}^m m_{t,y} - b_{t+2}^{s,m} - k_{t+2,m} - m_{t+1,m} \tag{4}$$

$$c_{t+2,o} = (R_t^l / \Pi_{t+2}^l)b_{t+1}^l + (R_{t+1}^s / \Pi_{t+2})b_{t+2}^{s,m} + r_{t+2}^k k_{t+2,m} + T_{t+2}^2 / P_{t+2} + r_{t+2}^m m_{t+1,m} \tag{5}$$

where τ_t is the rate of income tax, $\Pi_{t+1} = P_{t+1}/P_t$ is inflation between period t and t+1, and $\Pi^l_{t+2} = P_{t+2}/P_t$ is inflation between period t and t+2.

Consumers have CRRA preferences. They solve the following utility maximization problem:

$$\max_{\{c_{t,y}, c_{t+1,m}, c_{t+2,o}, a_{t+1,y}, a_{t+2,m}\}} U_t = \frac{1}{1-\gamma} E_t\left[c_{t,y}^{1-\gamma} + \beta c_{t+1,m}^{1-\gamma} + \beta^2 c_{t+2,o}^{1-\gamma}\right] \quad \text{s.t. (1) to (5)} \tag{6}$$

where $\gamma > 0$ is the coefficient of relative risk aversion, β is the private discount factor, and $a_{t+1,y}$ and $a_{t+2,m}$ are the vectors of asset holdings chosen by the young and middle-aged.

Letting $sdf_{t+1,m} \equiv \beta(c_{t+1,m}/c_{t,y})^{-\gamma}$ and $sdf_{t+1,o} \equiv \beta(c_{t+1,o}/c_{t,m})^{-\gamma}$, the first-order conditions are:[14]

$$1 = E_t[sdf_{t+1,m} r_{t+1}^k] \quad \text{for capital (when young), } k_y \tag{7}$$

$$1 = E_t[sdf_{t+1,m} r_{t+1}^s] \quad \text{for short-term debt (when young), } b^{s,y} \tag{8}$$

$$1 = E_t[sdf_{t+2}^l r_{t+2}^l] \quad \text{for long-term debt, } b^l \tag{9}$$

$$1 = E_t[sdf_{t+1,o} r_{t+1}^s] \quad \text{for short-term debt (when middle-aged), } b^{s,m} \tag{10}$$

$$1 = E_t[sdf_{t+1,o} r_{t+1}^k] \quad \text{for capital (when middle-aged), } k_m \tag{11}$$

where $sdf_{t+2}^l \equiv sdf_{t+2,o} sdf_{t+1,m}$, and $r_{t+1}^s \equiv R_t^s / \Pi_{t+1}$ and $r_{t+2}^l \equiv R_t^l / \Pi_{t+2}^l$ are real returns on debt.

2.2 Firms

The production sector consists of a representative firm which produces output using a Cobb-Douglas production function. The capital share of output is equal to α and the labour share is $1 - \alpha$. The firm hires capital and labour in competitive markets to maximise current profits. Total factor productivity, A, is stochastic and follows an AR(1) process: $\ln A_t = \rho_A \ln A_{t-1} + e_t$, where e_t is an IID-normal innovation with mean zero and standard deviation σ_e.

[14] In addition, there are two first-order conditions for money holdings relating to the reserve requirements in (1). See Section 1.A of the Supplementary Appendix for a derivation of the first-order conditions.

The real wage and the return on capital are given by

$$w_t = y_t - r_t^k k_t = (1-\alpha)A_t k_t^{\alpha} \quad (12)$$

$$r_t^k = \alpha y_t / k_t = \alpha A_t k_t^{\alpha-1} \quad (13)$$

2.3 Government

The government performs three functions. First, it levies an income tax on the young and middle-aged to fund a constant stream of government spending. Second, it commits to a money supply rule which is implemented through lump-sum monetary transfers to the middle-aged and old; see (2). Third, it sets the total supply of government bonds and the share of long-term debt. The analysis below considers the case where the government sets the share of long-term debt at levels seen in the data, as well as the case where the debt share is set optimally to maximise social welfare subject to the monetary policy regime in place.

The government budget constraint is given by

$$\tau_t w_t = g^* + (R_{t-2}^l / \Pi_t^l) b_{t-1}^l - b_{t+1}^l + (R_{t-1}^s / \Pi_t) b_t^{s,y} - b_{t+1}^{s,y} + (R_{t-1}^s / \Pi_t) b_t^{s,m} - b_{t+1}^{s,m} \quad (14)$$

where $g^* > 0$ is constant government spending per period.[15]

The government issues a fraction $0 \le v^l \le 1$ of long-term debt, so that $b^l = v^l b^{tot}$ is the supply of long-term debt and $b^{s,y} + b^{s,m} = (1 - v^l)b^{tot}$ is the total supply of short-term debt. This enables us to write the government budget constraint as follows:

$$\tau_t w_t = g^* + (R_{t-2}^l / \Pi_t^l) v^l b_{t-1}^{tot} + (R_{t-1}^s / \Pi_t)(1 - v^l) b_t^{tot} - b_{t+1}^{tot} \quad (15)$$

where v^l is the share of long-term debt and $1 - v^l$ is the share of short-term debt.[16]

This equation shows that the share of long-term debt influences the level of taxes necessary for the government to meet its spending commitments. The real returns on bonds are crucial because they determine the average cost of debt repayments on each type of debt. In turn, these returns depend crucially on the monetary policy regime in place through the inflation risk premium, which affects the level of the bonds yields R^s and R^l (see Appendix A).

The total supply of government debt is held constant as in Diamond (1965):[17]

$$b_t^{tot} = b^{tot} > 0 \quad (16)$$

The value of b^{tot} is chosen so that the equilibrium real interest rate in the deterministic steady-state is equal to some target value. The advantage of this approach is that it makes steady-state social welfare identical for all shares of long-term government debt. As a result, the optimal share of long-term debt under uncertainty is pinned solely down by the risk characteristics of the economy. The policy problem for the case where the share of long-term debt is chosen to maximize social welfare is set out in Section 4.

[15] This left side equation makes use of the fact that aggregate labour supply is normalized to 1, i.e. $L_y + L_m = 1$.
[16] Note that although the total supply of short-term debt is constant (because v^l and b^{tot} are constant), holdings of short-term debt by the young and middle-aged will vary over time.
[17] Strictly speaking, it is government debt per head that is constant in Diamond (1965). However, the distinction between aggregate and per capita values is irrelevant here because population is normalized to 1.

2.4 Market-clearing and equilibrium

Capital depreciates fully within a period, an assumption which is empirically reasonable given that each period lasts 20 years. Hence, aggregate investment is $i_t = k_{t+1} = k_{y,t+1} + k_{m,t+1}$.

Definition of equilibrium:[18]

A set of allocations and prices $\left\{c_{t,y}, c_{t,m}, c_{t,o}, b_t^{s,y}, b_t^{s,m}, b_t^l, k_{t,y}, k_{t,m}, m_{t,y}, m_{t,m}, R_t^s, R_t^l, r_t^k, \tau_t\right\}$ with the following properties for all t:

(1) Allocations $\left\{c_{t,y}, c_{t+1,m}, c_{t+2,o}, b_{t+1(d)}^{s,y}, b_{t+2(d)}^{s,m}, b_{t+1(d)}^l, k_{t+1,y}, k_{t+2,m}\ m_{t,y(d)}, m_{t+1,m(d)}\right\}$ solve the maximization problem of the young born in t and factors are paid their marginal products;

(2) The goods, money and bond markets clear:

$$
\begin{aligned}
& y_t = c_{t,y} + c_{t,m} + c_{t,o} + g* + k_{t+1} \\
& m_{t,y(d)} + m_{t,m(d)} = m_{t(s)} \\
& b_{t(d)}^{s,y} = b_{t(s)}^{s,y} \\
& b_{t(d)}^{s,m} = b_{t(s)}^{s,m} \\
& b_{t(d)}^l = b_{t(s)}^l = v^l b_{t(s)}^{tot} \\
& b_{t(s)}^{tot} = b_{t(s)}^{s,y} + b_{t(s)}^{s,m} + b_{t(s)}^l
\end{aligned}
$$

(3) The government budget constraint is satisfied:

$$\tau_t w_t = g* + (R_{t-2}^l / \Pi_t^l) b_{t-1(s)}^l - b_{t+1(s)}^l + (R_{t-1}^s / \Pi_t) b_{t(s)}^{s,y} - b_{t+1(s)}^{s,y} + (R_{t-1}^s / \Pi_t) b_{t(s)}^{s,m} - b_{t+1(s)}^{s,m}$$

(4) The reserve requirements on money holdings are binding: $m_{t,y(d)} = m_{t,m(d)} = \delta$

2.5 Social welfare

Social welfare is given by the expected discounted sum of lifetime utilities of all generations born from the initial period onwards:[19]

$$SW = (1-\omega) E\left[\sum_{t=0}^{\infty} \omega^t U_t\right] = E[U_t] \tag{17}$$

where $0 < \omega < 1$ is the social discount factor, and E is the unconditional expectations operator.

It is clear from (17) that the social discount factor ω will not affect social welfare. The social discount factor is therefore left unspecified.

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[18] Note that bracketed d and s subscripts are introduced in this section to denote demand and supply values. These subscripts are omitted in other sections of the paper in order to avoid unnecessary notation.

[19] This social welfare function ignores the utility of the initial old and middle-aged. However, Section 7.1 shows that this does not affect the baseline results. The second equality in (17) holds as long as U_t is stationary. In this paper, U_t and the other equations of the model are solved using a second-order perturbation method.

3 Monetary regimes and inflation

The government commits to money supply rules. Since it cannot avoid money supply shocks, it has imperfect control over inflation. The main difference between IT and PT lies in the way that monetary policy responds to inflationary shocks. Under IT, inflationary shocks are treated as 'bygones' and so have a permanent impact on the price level. Under PT, by contrast, inflationary shocks are undone in order to restore the price-level to its target path.

3.1 Inflation targeting

Under IT, the central bank is assumed to follow a money *growth* rule of the form:

$$M_t / M_{t-1} = \Pi^*(1+\varepsilon_{M,t}) \tag{18}$$

where $\varepsilon_{M,t}$ is a zero-mean IID-normal innovation with standard deviation σ_M, and Π^* is the target money supply growth rate, which is interpreted as an inflation target.

Since the reserve requirements on cash holdings are binding, money market equilibrium implies that $m_{t,y} + m_{t,m} = M_t / P_t = 2\delta$. Hence, inflation is given by:

$$\Pi_t = M_t / M_{t-1} = \Pi^*(1+\varepsilon_{M,t}) \tag{19}$$

Note that money supply shocks cause inflation to deviate from target. It is straightforward to show using (19) that the log price level follows a random walk with drift, so that the price level is non-stationary. In addition, expected inflation equals the inflation target: $E_{t-1}\Pi_t = \Pi^*$.

To see the implications of IT for holders of short and long-term debt, note that taking logs of (19) implies that short-term and long-term inflation risk are as follows:[20]

$$\text{var}_t(\ln \Pi_{t+1}) \approx \sigma_M^2 \tag{20a}$$

$$\text{var}_t(\ln \Pi_{t+2}^l) \approx 2\sigma_M^2 \tag{20b}$$

Since inflation risk is proportional to the forecast horizon under IT, long-term inflation risk is double short-term inflation risk. As a result, long-term bonds will be subject to greater revaluation risk than short-term bonds in an economy with inflation risk and IT.

3.2 Price-level targeting

Under PT, the central bank follows a money *level* rule of the form:

$$M_t = \Pi^*(1+\varepsilon_{M,t}) \tag{21}$$

Hence, inflation is given by:

$$\Pi_t = M_t / M_{t-1} = \Pi^*(1+\varepsilon_{M,t})/(1+\varepsilon_{M,t-1}) \tag{22}$$

This expression differs from that under IT due to the inclusion of a response to the past money supply innovation $\varepsilon_{M,t-1}$. The intuition is that the central bank must offset past money supply innovations in order to return the price level to its target path. It is easy to show using

[20] The focus here is on conditional inflation risk since this is what matters for the inflation risk premium.

(22) that the log price level is trend-stationary under PT, and that expected inflation is time-varying and equal to $E_{t-1}\Pi_t = \Pi^*/(1+\varepsilon_{M,t-1})$.

To see the implications of PT for holders of short and long-term debt, note that (22) implies that short-term and long-term inflation risk are equal:

$$\mathrm{var}_t(\ln \Pi_{t+1}) = \mathrm{var}_t(\ln \Pi^l_{t+2}) \approx \sigma^2_M \tag{23}$$

Equation (23) tells us that inflation risk does not rise with the forecast horizon under PT, in contrast to the situation under IT. Long-term bonds will thus be less risky in real terms under a PT regime because revaluation risk due to unanticipated inflation is lower.

4 Share of long-term debt

This section first considers the implications of the share of long-term debt for life-cycle consumption. It then sets out the policy problem of the government when it chooses the share of long-term debt optimally and discusses how this problem is solved numerically.

4.1 Implications for life-cycle consumption

To better understand the role of short and long-term government debt, it is instructive to write the generational budget constraints (3) to (5) in a way that shows their dependence on the total supply of bonds and the share of long-term debt v^l. Substituting for taxes using (16), money holdings using (1), monetary transfers using (2), and noting that $b^l = v^l b^{tot}$ where b^{tot} is constant, we have:

$$c_{t,y} = w_t L_y - L_y g^* - v^l (R^l_{t-2}/\Pi^l_t) b^{tot} - (1-v^l)(R^s_{t-1}/\Pi_t) b^{tot} + b^{s,m}_{t+1} - k_{t+1,y} - \delta \tag{24}$$

$$c_{t+1,m} = w_{t+1} L_m - L_m g^* + r^k_{t+1} k_{t+1,y} - [v^l (R^l_{t-1}/\Pi^l_{t+1}) b^{tot} + (R^s_t/\Pi_{t+1}) b^{s,m}_{t+1}] - b^{s,m}_{t+2} - k_{t+2,m} \tag{25}$$

$$c_{t+2,o} = (R^l_t/\Pi^l_{t+2}) v^l b^{tot} + (R^s_{t+1}/\Pi_{t+2}) b^{s,m}_{t+2} + r^k_{t+2} k_{t+2,m} + \delta \tag{26}$$

where Π_t and $\Pi^l_t = \Pi_t \Pi_{t-1}$ depend on the monetary policy regime that is implemented.

Written this way, it is clear that changing the share of long-term debt v^l will affect consumption over the life-cycle. For instance, if the average real return on long-term debt exceeds that on short-term debt, lowering v^l will tend to raise average consumption when young (because it implies lower taxes). Likewise, Equation (25) shows that consumption by the middle-aged will tend to rise as v^l increases, while Equation (26) shows that average consumption by the old will tend to fall because holdings of long-term debt will be lower.

4.2 Optimal share of long-term debt

The government's problem is to choose the share of long-term government debt that maximises social welfare, subject to the monetary policy regime in place and the equilibrium conditions of the model. As discussed below, the model is solved using a second-order perturbation method.

The policy problem of the government can be stated as follows:

$$\max_{v^l \in [0,1]} SW = E[U_t]$$

subject to (7)-(16), (24)-(26), market-clearing and

$$\Pi_t = \begin{cases} \Pi^*(1+\varepsilon_{M,t}) & \text{under IT} \\ \Pi^*(1+\varepsilon_{M,t})/(1+\varepsilon_{M,t-1}) & \text{under PT} \end{cases} \tag{27}$$

The debt share that solves (27) was computed numerically using a second-order perturbation method in Dynare++ (Adjemian et al., 2011). In particular, social welfare was computed for a large number of long-term shares, v^l, in the interval [0,1] with the aid of an algorithm provided on Wouter Den Haan's personal webpage.[21]

To understand the numerical results that follow, it is helpful to consider a second-order expansion of social welfare around $c_{t,y} = E[c_{t,y}]$, $c_{t+1,m} = E[c_{t+1,m}]$ and $c_{t+2,o} = E[c_{t+2,o}]$: [22]

$$SW \approx \frac{1}{1-\gamma}\left[E[c_y]^{1-\gamma} + \beta E[c_m]^{1-\gamma} + \beta^2 E[c_o]^{1-\gamma}\right] - \frac{1}{2}\left[|U_{11}|\operatorname{var}[c_y] + |U_{22}|\operatorname{var}[c_m] + |U_{33}|\operatorname{var}[c_o]\right] \tag{28}$$

where subscripts denote partial derivatives of the social welfare function to each argument, evaluated at unconditional mean consumption levels: $E[c_y]$, $E[c_m]$, $E[c_o]$.

This expression shows that social welfare increases with mean consumption levels and falls with consumption variances (due to risk aversion). Consequently, the optimal share of long-term debt can be understood in terms of these unconditional moments. The numerical analysis that follows therefore reports these moments and focuses on their determinants.

5 Calibration

The model is roughly calibrated to the UK economy. In particular, the parameters are chosen to roughly match key ratios in the data. As steady-state ratios depend upon several different parameters, the baseline calibration uses parameter values which are plausible and give good overall performance against target ratios.[23] The baseline calibration is listed in Table 1.

Table 1 – Baseline calibrated values

α	Capital share	0.24	g^*	Government spending per period	0.085
β	Private discount factor	0.54	Π^*	Inflation target	1.486
γ	Risk aversion	2.5	ρ_A	TFP persistence	0
δ	Real money holdings (m_y, m_m)	0.01	σ_e, σ_M	Std(e_t), Std ($\varepsilon_{M,t}$)	0.05
L_y	Labour supply (young)	0.55	L_m	Labour supply (middle-age)	0.45
b^{tot}	Total supply of govt. debt	0.01	v^l	Share of long-term debt	0.50

5.1 Aggregate uncertainty

The model contains two aggregate shocks: a productivity shock and a money supply shock. Persistence in the productivity shock was set at zero as in Olovsson (2010) because there is no convincing empirical evidence that productivity is persistent over generational horizons. Assuming positive persistence would not overturn the baseline results and is considered as a robustness check in sensitivity analysis; see Section 7.3.

[21] The author is grateful to Wouter Den Haan for making his codes publicly available.
[22] This expression makes use of the fact, that under stationarity: $E[x_{t+i}] = E[x_t]$ and $\operatorname{var}[x_{t+i}] = \operatorname{var}[x_t]$.
[23] In some models, steady-state ratios are pinned down by a single parameter so that calibrated values can be set to match target ratios exactly. This is not the case for most of the key ratios in this model.

It is assumed that both shock innovations have the same standard deviation of 0.05. This relatively high standard deviation reflects the fact that these are shocks at a *generational horizon*. The calibration of the productivity innovation standard deviation is similar to that in Hatcher (2014) in an OG model with 20 years per period. The money supply innovation standard deviation was set at 0.05 because this implies that inflation has a standard deviation of 7.4%.[24] By comparison, the standard deviation of 20-year UK inflation is around 7% based on the Consumer Prices Index (CPI) over the period 1988 to 2014.

5.2 Preference and other parameters

The discount factor β is set at 0.54, which is equivalent to an annual value of 0.97. The coefficient of relative risk aversion is set at 2.5. The parameter α was set at 0.24, implying a capital income share of 24%. This is slightly lower than standard calibrations but helps the model to get close to the target ratios of investment and tax revenue to GDP (see Table 2). Labour supply by the young is set equal to 0.55, which implies that labour supply by the middle-aged is 0.45. This parameterization was chosen to ensure that the disposable income of the young was not implausibly low given that they do not receive any asset income.

Government spending per period g^* was set at 0.085, which implies a government spending-GDP share of 15% of GDP in the calibrated model. The inflation target Π^* was set at 1.486, which is consistent with trend inflation of 2% per year. The reserve parameter δ was set at 0.01 to match the UK share of notes and coins in GDP. For the purpose of calibration, the share of long-term debt v^l was set at 0.5, or 50%.[25] Given these calibrated values, the total supply of government debt b^{tot} was set at 0.01 since this implies a steady-state real interest rate of 1.5, which is equivalent to an annual interest rate of 2% per annum.

5.3 Model solution and key ratios

Table 2 – Target versus model ratios

Ratio	Target	Definition	Deterministic	Stochastic	Notes
$\tau w / y$	0.14	Income tax revenue/GDP	0.17	0.17	Target: UK (HMT)
i / y	0.17	Investment/GDP	0.16	0.16	Target: UK (ONS)
c / y	0.65	Consumption/GDP	0.69	0.69	Target: UK (ONS)
m / y	0.04	Currency/GDP	0.04	0.04	Target: UK (ONS)

Notes: the model and target government spending ratios are given by $g/y = 1 - c/y - i/y$.

The calibrated model does fairly well against target ratios (see Table 2).[26] The UK investment-GDP share averaged 17% over the period 2005-2013 and over the same period the consumption-GDP share was around 65% (ONS, 2014), implying target ratios of 0.17 and 0.65. The calibrated model gives ratios of 0.16 and 0.69. The GDP share of notes and coins in

[24] The qualitative conclusions of the model are not sensitive to the assumed value of the standard deviations. However, as discussed in Section 6.3, excluding productivity risk from the model has important implications.
[25] The UK Debt Management Office (DMO) classifies all bonds with maturities exceeding 15 years as 'long-term'. According to DMO (2014, Table C2), nominal debt with maturities less 15 years has accounted for around one-half of the gilt portfolio over the past decade, so $v^l = 0.5$ was chosen as the baseline calibration.
[26] The results in Table 2 relate to an IT regime with a long-term debt share of 0.5. The results reported are not sensitive to the calibrated debt share.

the model matches the UK share of 4% over the decade to 2010; see ONS (2011). Lastly, the ratio of income tax revenue to GDP is close to the UK share of receipts from income and wealth taxes, which averaged around 14% of GDP from 2000 to 2012 (HM Treasury, 2013).

6 Results

The model was solved using a second-order perturbation in Dynare++ (Adjemian et al., 2011).[27] The analysis in this section begins by comparing the performance of IT and PT in an economy in which the share of long-term government debt is calibrated to roughly match the share in the data. It then turns to the case where the share of long-term debt is chosen optimally before considering the importance of productivity risk for these results.

6.1 Calibrated debt share

The share of long-term government debt was set at $v^l = 0.5$ under both IT and PT, implying that one half of government debt is short-term (i.e. 1-period debt) and that the other half is long-term (i.e. 2-period debt). As discussed above, this calibration is intended to be representative of the UK economy where both short and long-term debt are issued.

The results are reported in Table 3. The welfare gain (or loss) from PT is defined as the fractional increase (reduction) in aggregate consumption, λ, necessary to equate social welfare under IT and PT: $SW^{IT}(1+\lambda)^{1-\gamma} = SW^{PT}$. As well as the baseline case where $v^l = 0.5$, results are also shown for the cases where long-term debt accounts for one-quarter and three-quarters of the total stock of debt: i.e. $v^l = 0.25$ and $v^l = 0.75$.

Table 3 – IT vs PT in an economy with both short and long-term debt

Share of long-term debt v^l	**Welfare gain of PT** (λ, % of consumption)
0.25	0.005
0.50	**0.009**
0.75	0.011

Relative to an IT regime, PT implies a modest welfare gain. Under the baseline calibration where $v^l = 0.5$, the welfare gain is around 0.01% of aggregate consumption, and the results are similar for the cases of $v^l = 0.25$ and $v^l = 0.75$. PT raises social welfare because it lowers long-term inflation risk relative to IT, so that long-term debt is less risky. As a result, the inflation risk premium on long-term debt is lower under PT, which lowers the average real return payable on long-term debt. This, in turn, means that taxes are lower, which implies an increase in mean consumption by the young and middle-aged. In addition to this, the lower riskiness of the real return on long-term debt means that the volatility of taxes is lower under PT, so that consumption risk is reduced for the young and middle-aged.

One reason that PT delivers only modest welfare gains is that average consumption by the old is lower than under IT. This is because the old receive a higher a return on long-term debt under IT due to the higher inflation risk premium in nominal interest rates. In this respect, PT

[27] The baseline model was randomly simulated 100 times for 1100 periods (with a 'burn in' of 100 periods), giving a total of 100,000 simulated values to calculate unconditional moments.

has distributional implications: it raises consumption for workers (the young and middle-aged) but lowers consumption by the retired old. In addition to this, PT also raises consumption risk for the old, albeit only marginally.

Overall, these results suggest that PT would deliver modest welfare gains relative to IT in an economy where both short and long-term debt are issued, as in the data. It remains to be seen, however, whether PT will outperform IT in an economy where the share of long-term debt is set optimally. This question is taken up in the next section.

6.2 Optimal share of long-term debt

The optimal share of long-term debt under each regime is computed as described in Section 4.2. The welfare gain (loss) of issuing long-term debt is computed as the fractional increase (reduction) in aggregate consumption, λ_v, that equates social welfare under a zero long-term debt share with that when the long-term share is v^l. Hence, $SW_{(0)}\,(1+\lambda_v)^{1-\gamma} = SW_{(v^l)}$. To shed light on the welfare results, the unconditional moments of real variables in the model are reported, including the unconditional moments of consumption that matter for social welfare; see Equation (28). The results are reported in Figures 1 to 3.

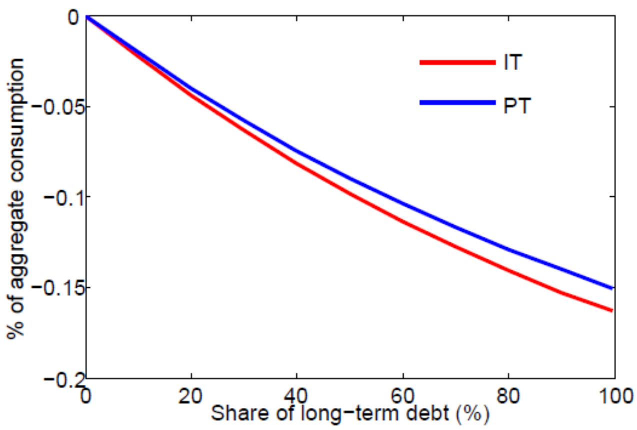

Fig 1 – Social welfare gain relative to the case of zero long-term debt

Figure 1 shows that social welfare falls under both IT and PT as the share of long-term debt is increased. As a result, it is optimal to issue only short-term debt under both regimes. In addition, the welfare implications of the share of long-term debt are non-trivial. For instance, moving from an economy with only short-term debt to one in which half of debt is long-term implies a welfare loss of almost 0.10% of aggregate consumption.

It is optimal to issue only short-term debt under IT and PT because taxes rise as the share of long-term debt is increased. This happens because real returns on both short and long-term debt increase as the share of long-term debt rises. This, in turn, is a result of the aggregate demand for capital falling as the young and middle-aged substitute towards short-term bonds in order to lessen consumption risk. Consumption risk faced by the old and middle-aged increases with the share of long-term debt due to the fact that the real return on long-term debt is much more volatile than the return on short-term debt as a result of productivity risk causing larger fluctuations in long-term *ex ante* real interest rates than short-term *ex ante* real interest rates (see Footnote 30).

Figures 1 to 3 show that when the share of long-term debt is set at its optimal value of zero, IT and PT have essentially identical implications for consumption and social welfare. The reason is simply that short-term inflation risk is the same under both regimes (see equations (20) and (23)) and expected inflation, which does differ under IT and PT, does not affect real bonds returns or optimal asset holdings because nominal debt compensates bondholders for *expected* changes in inflation through movements in nominal interest rates. Thus, there is nothing to choose between the IT and PT if the share of long-term debt is set optimally. This finding provides a counterargument to the conventional wisdom that PT would deliver welfare gains because it lowers long-term inflation risk.[28]

It is important to note, however, that PT outperforms IT whenever long-term debt is issued. The reason is that PT raises consumption by the young and middle-aged since it implies lower average taxes than IT, due to average real returns on bonds being lower under PT. This, in turn, is a result of the inflation risk premium on short and long-term debt being lower under PT. The inflation risk premium on short-term debt is lower under PT because it lowers consumption risk for the middle-aged relative to IT, whereas the inflation risk premium on long-term debt is lower due to the reduction in long-term inflation risk under a PT regime.[29] PT also lowers consumption risk for the young and middle-aged in an economy with long-term debt, because it reduces the variance of taxes. Consistent with the results reported in Table 3, the potential welfare gains from PT rise as the share of long-term debt in the government bond portfolio is increased.

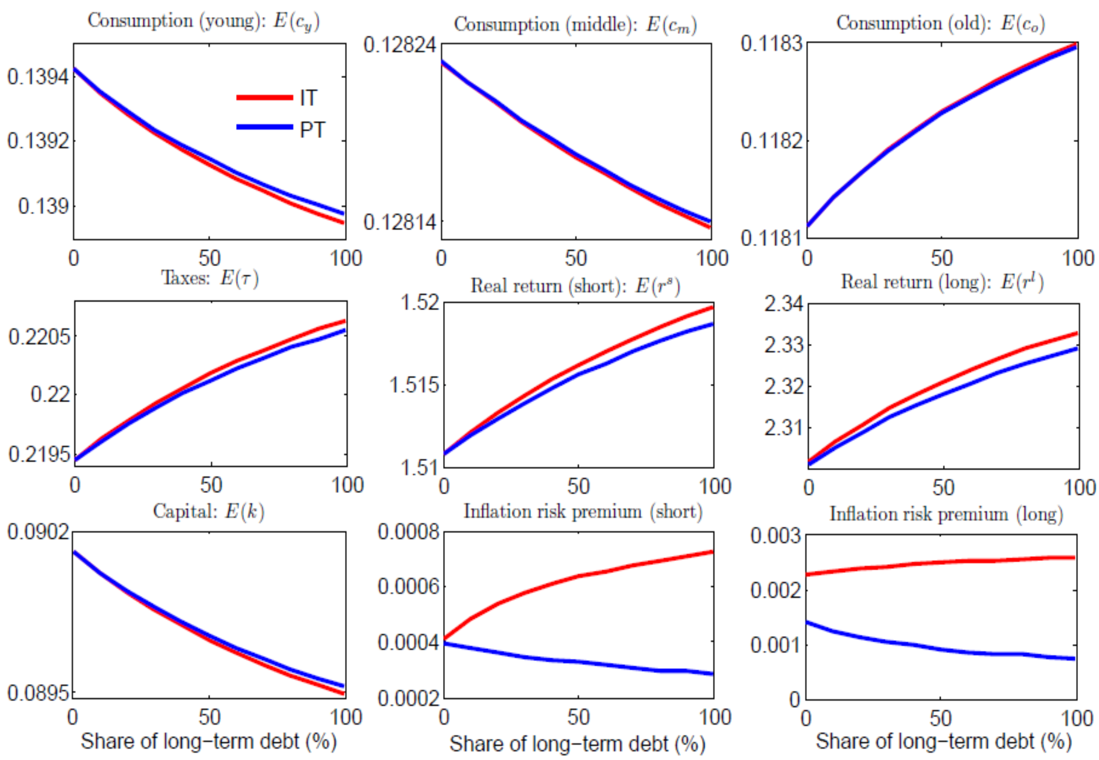

Fig 2 – Share of long-term debt and means of real variables

[28] It is shown in Section 7.3 that this conclusion hinges on the presence of productivity risk.
[29] Expressions for the inflation risk premium are given in Appendix A.

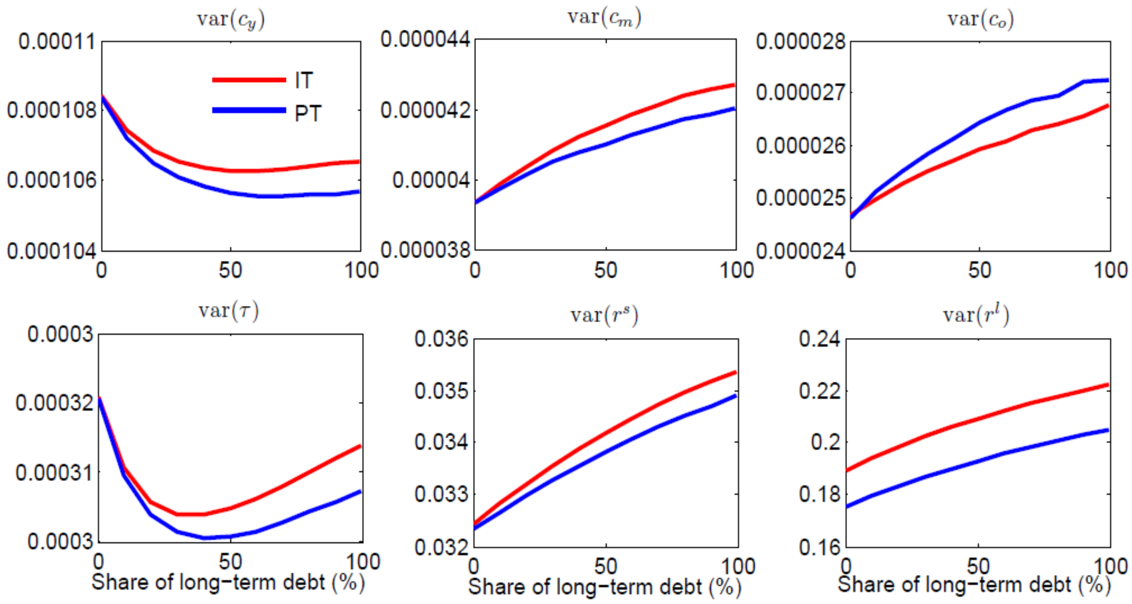

Fig 3 – Share of long-term debt and variances of real variables

6.3 Importance of productivity risk

To illustrate the importance of productivity risk for the above results, this section considers a version of the model in which there are no money supply shocks, so that inflation risk is absent. Since IT and PT differ only in their response to inflationary shocks, there is no distinction between the two regimes in this version of the model. As a result, this analysis is able to isolate the impact of productivity risk on the optimal share of long-term debt. The results are reported in Figures 4 and 5.

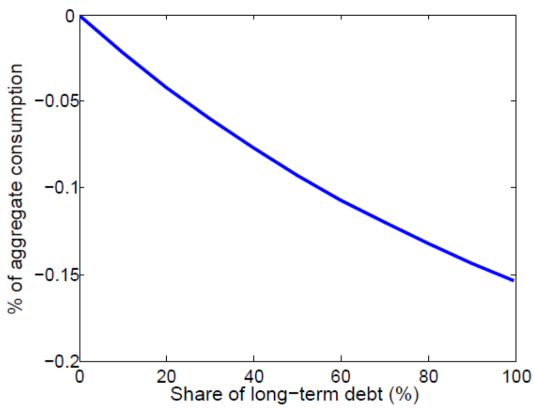

Fig 4 – Social welfare gain relative to the case of zero long-term debt (no inflation risk)

When productivity shocks are the only source of risk in the model, it is optimal to issue only short-term debt and the welfare losses associated with issuing long-term debt are quantitatively quite similar to those in the baseline model (compare Figures 1 and 4). Thus, productivity risk is crucial for the result that issuing only short-term debt is optimal. Figure 5 shows why productivity risk is crucial. The key point is that productivity risk implies volatility in the real returns on nominal bonds because it leads to fluctuations in the nominal interest rates R^s and R^l by making *ex ante* real interest rates vary over time.[30] These fluctuations are not a source of risk for agents in the model (because they are forecastable at

[30] Short and long ex ante real rates are given by $r_{t,f,short} = 1/E_t[sdf_{t+1,m}]$ and $r_{t,f,long} = 1/E_t[sdf_{t+2,o}\ sdf_{t+1,m}]$.

date *t*) but they reduce social welfare because they mean that consumption levels vary more from one generation to the next; see Equation (28).

Short-term *ex ante* real rates are relatively stable because productivity shocks induce a strong correlation between consumption today and expected consumption one period ahead. This is because holdings of short-term bonds and capital will tend to fall (rise) when there is a positive (negative) productivity shock. Long-term rates are much more volatile, however, because the correlation between consumption today and expected consumption in two periods ahead (i.e. in old age) is much weaker since agents know that their holdings of short-term bonds and capital will be reoptimized in middle-age. Hence, expected consumption in old age and consumption when young will tend to move by different amounts in response to productivity shocks, implying rather volatile long-term interest rates. It is this volatility that drives the result that issuing only short-term debt is optimal.

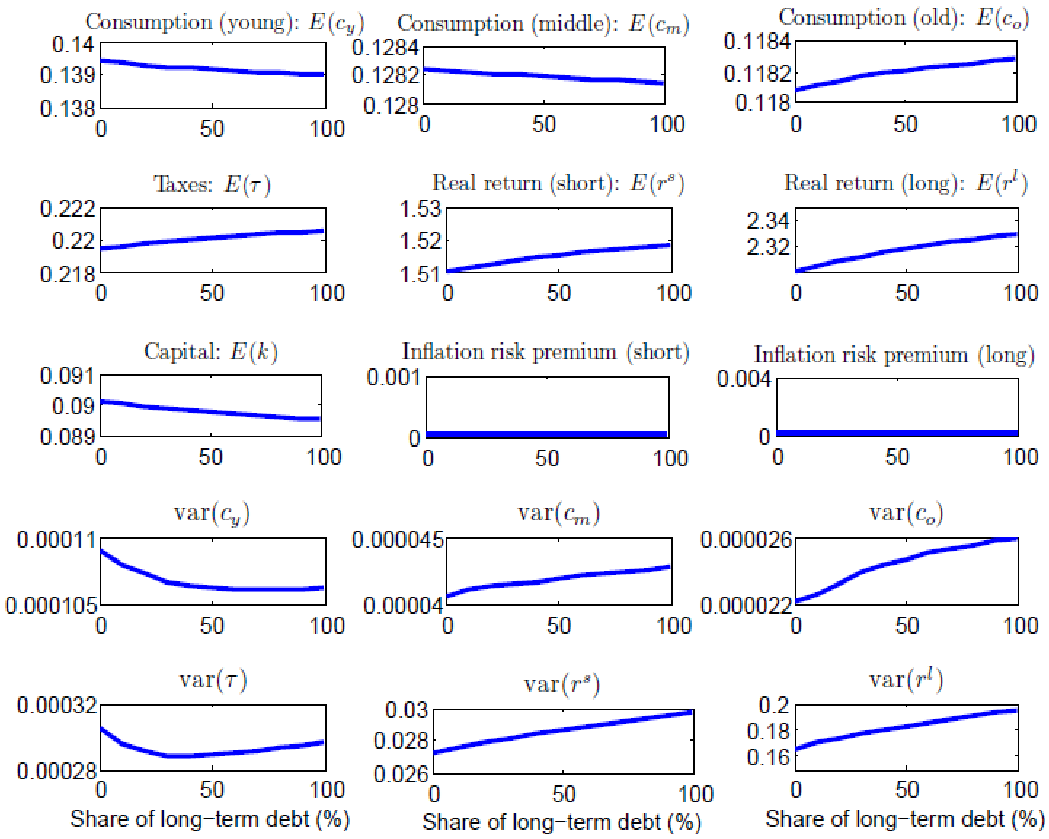

Fig 5 – Share of long-term debt and real variables (no inflation risk)

The importance of productivity risk can also be seen by considering a version of the model in which productivity shocks are eliminated, so that only inflation risk remains. In this version of the model, issuing only short-term debt remains the optimal policy under IT, but not under PT (see Section 2A of the Supplementary Appendix). As a result, the equivalence of IT and PT disappears, with PT implying modest welfare gains relative to IT even if the share of long-term debt is set optimally. It is important to note, however, that this result disappears if even a small amount of productivity risk is introduced. It therefore does not appear to hold important practical implications for the choice between IT and PT regimes.

7 Robustness checks

This section investigates whether the results from the baseline model are robust. It begins by considering the treatment of utility of initial generations before turning to parameter sensitivity analysis.

7.1 Utility of the initial old and middle-aged

In the baseline model, social welfare is given by the discounted sum of lifetimes utilities of all generations born in the first period onwards. A welfare function of this form ignores the utility of the initial old and initial middle-aged. However, including the utility of the initial old and middle-aged in the social welfare function does not affect the main conclusions.[31]

Table 6 and Figure 7 report the social welfare results for this case. It is notable that although the welfare losses associated with issuing long-term debt are lower than in the baseline model, the potential welfare gains of PT in an economy with long-term debt remain in intact, albeit that they are smaller. Moreover, it is still optimal to issue only short-term debt under both IT and PT, so that the potential welfare gains of PT are eliminated in this case.

Table 6 – IT vs PT in an economy with both short and long-term debt (amended SWF)

Share of long-term debt v^l	**Welfare gain of PT** (λ, % of consumption**)**
0.25	0.002
0.50	**0.004**
0.75	0.005

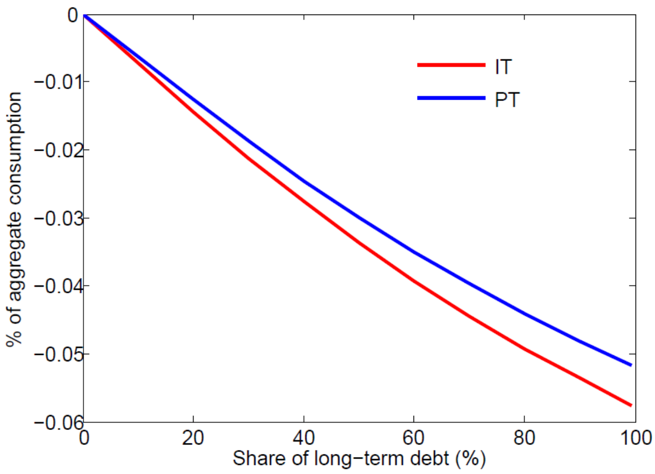

Fig 7 – Social welfare gain relative to the case of zero long-term debt (amended SWF)

7.2 Parameter sensitivity analysis

Experimentation with ‘high’ and ‘low’ calibrations of model parameters (listed in Section C of the Supplementary Appendix) did not overturn the main conclusions regarding the optimal

[31] The amended social welfare function is derived in Section 2.B of the Supplementary Appendix.

share of long-term debt under IT and PT or the result that PT raises social welfare when some long-term government debt is issued.[32]

8 Conclusion

This paper has compared inflation targeting (IT) and price-level targeting (PT) in an economy with short and long-term nominal government debt. The analysis was motivated by the fact that these two regimes have very different implications for long-term inflation risk. Under IT, inflation risk increases with the forecast horizon, because there is base-level drift in the price level. Under PT, by contrast, inflation risk does not change with the forecast horizon, so that the purchasing power of long-term nominal claims is relatively stable. The analysis proceeded in a simple overlapping generations (OG) model in which households live for three periods: youth, middle-age and old age. The model is well-suited for a comparison of IT and PT because both short and long-term nominal debt are present and the model permits the share of long-term nominal debt to be calibrated or chosen optimally.

The main finding is that PT raises social welfare relative to IT in an economy in which both short and long-term debt are issued, as in the data. However, if the share of long-term debt is chosen optimally to maximise social welfare, only short-term debt should be issued under both IT and PT, so that the result that the potential welfare gains of PT are eliminated. These results are robust to model parameterization and other robustness checks. However, the result that issuing only short-term debt is optimal hinges on the presence of productivity risk. If productivity risk is absent, it is optimal to issue only long-term debt under IT, but not under PT. In this case, there are welfare gains to PT even if the share of long-term is set optimally. Crucially, however, this result disappears if even a small amount of productivity risk is introduced into the model.

These results suggest that while there could be long-term welfare gains of adopting PT, this will depend crucially on whether the composition of the government debt portfolio is chosen to maximise social welfare. The main message of the paper is that the welfare gains of PT are likely to be smaller when the share of long-term government debt is set optimally or in a near-optimal manner. This conclusion casts doubt on the conventional wisdom that PT would raise social welfare because it lowers long-term inflation risk.

[32] The results are available from the author on request.

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Appendix

Appendix A – The inflation risk premium

This Appendix shows how the inflation risk premiums on short and long-term debt are defined.

The first-order conditions for bond holdings imply that

$$1 = E_t[sdf_{t+1,m}]E_t[r^s_{t+1}] + \mathrm{cov}_t[sdf_{t+1,m}, r^s_{t+1}] \tag{A1}$$

$$1 = E_t[sdf^l_{t+2}]E_t[r^l_{t+2}] + \mathrm{cov}_t[sdf^l_{t+2}, r^l_{t+2}] \tag{A2}$$

The inflation risk premium on short-term debt

In order to derive an expression for the inflation risk premium on short debt, note that the riskless one-period real interest rate can be defined as follows:

$$1 = r_t^{s,f} E_t[sdf_{t+1,m}] \tag{A3}$$

Hence, using (A1), the difference in expected real returns on nominal and real short-term debt is

$$E_t[r^s_{t+1}] - r_t^{s,f} = -\frac{\mathrm{cov}_t[sdf_{t+1,m}, r^s_{t+1}]}{E_t[sdf_{t+1,m}]} = -\frac{R^s_t \,\mathrm{cov}_t[sdf_{t+1,m}, \Pi^{-1}_{t+1}]}{E_t[sdf_{t+1,m}]} \tag{A4}$$

The expression in (A5) is the inflation risk premium on short-term debt. It it is non-zero unless (i) consumers are risk-neutral, or (ii) short-term inflation risk is zero.

The inflation risk premium can be written in terms of inflation risk and consumption risk using (A4):

$$E_t[r^s_{t+1}] - r_t^{s,f} = -\frac{R^s_t \mathrm{corr}_t[sdf_{t+1,m}, \Pi^{-1}_{t+1}]\sqrt{\mathrm{var}_t[sdf_{t+1,m}]}\sqrt{\mathrm{var}_t[\Pi^{-1}_{t+1}]}}{E_t[sdf_{t+1,m}]} \tag{A5}$$

The inflation risk premium on long-term debt

In order to derive an expression for the inflation risk premium on short debt, note that the riskless two-period real interest rate can be defined as follows:

$$1 = r_t^{l,f} E_t[sdf^l_{t+2}] \tag{A6}$$

Hence, using (A3), the difference in expected real returns on nominal and real long-term debt is

$$E_t[r^l_{t+2}] - r_t^{l,f} = -\frac{\mathrm{cov}_t[sdf^l_{t+2}, r^l_{t+2}]}{E_t[sdf^l_{t+2}]} = -\frac{R^l_t \,\mathrm{cov}_t[sdf^l_{t+2}, (\Pi^l_{t+2})^{-1}]}{E_t[sdf^l_{t+2}]} \tag{A7}$$

The inflation risk premium can be written in terms inflation risk and consumption risk using (A8):

$$E_t[r^l_{t+2}] - r_t^{l,f} = -\frac{R^l_t \mathrm{corr}_t[sdf^l_{t+2}, (\Pi^l_{t+2})^{-1}]\sqrt{\mathrm{var}_t[sdf^l_{t+2}]}\sqrt{\mathrm{var}_t[(\Pi^l_{t+2})^{-1}]}}{E_t[sdf^l_{t+2}]} \tag{A8}$$

The expression in (A9) is the inflation risk premium on long-term debt. It it is non-zero unless (i) consumers are risk-neutral, or (ii) long-term inflation risk is zero.

Supplementary Appendix (For Online Publication Only)

Section 1 – Derivations and proofs

1.A – Derivation of first-order conditions (baseline model)

Consumers solve a maximization problem of the form

$$\max_{\{c_{t,y},c_{t+1,m},c_{t+2,o},a_{t+1,y},a_{t+2,m}\}} U_t = \frac{1}{1-\gamma} E_t\left[c_{t,y}^{1-\gamma} + \beta c_{t+1,m}^{1-\gamma} + \beta^2 c_{t+2,o}^{1-\gamma}\right] \quad \text{(A1)}$$

subject to

$$c_{t,y} = w_t(1-\tau_t)L_y - b_{t+1}^l - b_{t+1}^{s,y} - k_{t+1,y} - m_{t,y} \quad \text{(Young)}$$

$$c_{t+1,m} = w_{t+1}(1-\tau_{t+1})L_m + r_{t+1}^k k_{t+1,y} + r_{t+1}^s b_{t+1}^{s,y} + T_{t+1}^1 / P_{t+1} + r_{t+1}^m m_{t,y} - b_{t+2}^{s,m} - k_{t+2,m} - m_{t+1,m} \quad \text{(Middle)}$$

$$c_{t+2,o} = r_{t+2}^l b_{t+1}^l + r_{t+2}^s b_{t+2}^{s,m} + r_{t+2}^k k_{t+2,m} + T_{t+2}^2 / P_{t+2} + r_{t+2}^m m_{t+1,m} \quad \text{(Old)}$$

$$m_{t,y},\ m_{t,m} \geq \delta \quad \text{(Reserve requirements on money)}$$

where $r_{t+1}^s \equiv R_t^s / \Pi_{t+1}$, $r_{t+2}^l \equiv R_t^l / \Pi_{t+2}^l$, and the vectors of assets chosen by households when young and middle-aged are $a_{t+1,y} \equiv (k_{t+1,y}, b_{t+1}^l, b_{t+1}^{s,y}, m_{t,y})$ and $a_{t+2,m} \equiv (k_{t+2,m}, b_{t+2}^{s,m}, m_{t+1,m})$, respectively.

The Lagrangian for this problem is

$$L_t = E_t \begin{Bmatrix} U_t + \lambda_{t,y}[w_t(1-\tau_t)L_y - b_{t+1}^l - b_{t+1}^{s,y} - k_{t+1,y} - m_{t,y} - c_{t,y}] + \mu_{t,y}[m_{t,y} - \delta] \\ \lambda_{t+1,m}[w_{t+1}(1-\tau_{t+1})L_m + r_{t+1}^k k_{t+1,y} + r_{t+1}^s b_{t+1}^{s,y} + T_{t+1}^1 / P_{t+1} + r_{t+1}^m m_{t,y} - b_{t+2}^{s,m} - k_{t+2,m} - m_{t+1,m} - c_{t+1,m}] \\ + \mu_{t+1,m}[m_{t+1,m} - \delta] + \lambda_{t+2,o}[r_{t+2}^l b_{t+1}^l + r_{t+2}^s b_{t+2}^{s,m} + r_{t+2}^k k_{t+2,m} + T_{t+2}^2 / P_{t+2} + r_{t+2}^m m_{t+1,m} - c_{t+2,o}] \end{Bmatrix} \quad \text{(A2)}$$

First-order conditions are as follows:[33]

$$c_{t,y} : \partial U_t / \partial c_{t,y} = \lambda_{t,y}, \quad c_{t+1,m} : \partial U_t / \partial c_{t+1,m} = \lambda_{t+1,m} \quad c_{t+2,o} : \partial U_t / \partial c_{t+2,o} = \lambda_{t+2,o},$$
$$k_{t+1,y} : \lambda_{t,y} = E_t[\lambda_{t+1,m} r_{t+1}^k], \quad b_{t+1}^l : \lambda_{t,y} = E_t[\lambda_{t+2,o} r_{t+2}^l], \quad b_{t+1}^{s,y} : \lambda_{t,y} = E_t[\lambda_{t+1,m} r_{t+1}^s],$$
$$k_{t+2,m} : \lambda_{t+1,m} = E_{t+1}[\lambda_{t+2,o} r_{t+2}^k], \quad b_{t+2}^{s,m} : \lambda_{t+1,m} = E_{t+1}[\lambda_{t+2,o} r_{t+2}^s],$$
$$m_{t,y} : \lambda_{t,y} = E_t[\lambda_{t+1,m} r_{t+1}^m] + \mu_{t,y}, \quad m_{t+1,m} : \lambda_{t+1,m} = E_{t+1}[\lambda_{t+2,O} r_{t+2}^m] + \mu_{t+1,m},$$

By substitution, this system can be reduced to seven Euler equations:

$$\frac{\partial U_t}{\partial c_{t,y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,m}} r_{t+1}^k\right], \quad \frac{\partial U_t}{\partial c_{t,y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,m}} r_{t+1}^s\right], \quad \frac{\partial U_t}{\partial c_{t,y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+2,o}} r_{t+2}^l\right], \quad \frac{\partial U_t}{\partial c_{t,m}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,o}} r_{t+1}^k\right],$$
$$\frac{\partial U_t}{\partial c_{t,m}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,o}} r_{t+1}^s\right], \quad \frac{\partial U_t}{\partial c_{t,y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,m}} r_{t+1}^m\right] + \mu_{t,y}, \quad \frac{\partial U_t}{\partial c_{t,m}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,o}} r_{t+1}^m\right] + \mu_{t,m}$$

[33] First-order conditions involving expectations at time t+1are implied by taking the expectations operator E_{t+1} through both sides of the equation to account for the fact that the middle-aged have access to this information set when choosing their asset holdings.

The partial derivatives of U_t are given by:

$$\partial U_t / \partial c_{t,y} = c_{t,y}^{-\gamma}, \qquad \partial U_t / \partial c_{t+1,m} = c_{t+1,m}^{-\gamma}, \qquad \partial U_t / \partial c_{t+2,o} = c_{t+2,o}^{-\gamma}$$

Letting $sdf_{t+1,m} \equiv \beta(c_{t+1,m} / c_{t,y})^{-\gamma}$, $sdf_{t+1,o} \equiv \beta(c_{t+1,o} / c_{t,m})^{-\gamma}$ and $sdf_{t+2}^{l} \equiv sdf_{t+2,o} sdf_{t+1,m}$ gives the Euler equations reported in the main text, plus two for money holdings ((A8) and (A9)):

$$1 = E_t[sdf_{t+1,m} r_{t+1}^{k}] \tag{A3}$$
$$1 = E_t[sdf_{t+1,m} r_{t+1}^{s}] \tag{A4}$$
$$1 = E_t[sdf_{t+2}^{l} r_{t+2}^{l}] \tag{A5}$$
$$1 = E_t[sdf_{t+1,o} r_{t+1}^{s}] \tag{A6}$$
$$1 = E_t[sdf_{t+1,o} r_{t+1}^{k}] \tag{A7}$$
$$1 = E_t[sdf_{t+1,m} r_{t+1}^{m}] + \tilde{\mu}_{t,y} \tag{A8}$$
$$1 = E_t[sdf_{t+1,o} r_{t+1}^{m}] + \tilde{\mu}_{t,m} \tag{A9}$$

where $\tilde{\mu}_{t,y} \equiv \mu_{t,y} / \lambda_{t,y}$ and $\tilde{\mu}_{t,m} \equiv \mu_{t,m} / \lambda_{t,m}$.

1.B – The binding legal constraint on money holdings

Proposition: The constraint binds with strict equality when $R_t^s > 1$

Proof.

By equations (A4) and (A8), the Lagrange multiplier on the cash constraint on the young is given by

$$\mu_{t,y} = \lambda_{t,y} E_t[sdf_{t+1,m}(r_{t+1}^{s} - r_{t+1}^{m})] \tag{B1}$$

Since the real return on one-period nominal bonds is $r_{t+1}^{s} \equiv R_t^s / \Pi_{t+1} = R_t^s r_{t+1}^{m}$, we can say that

$$\mu_{t,y} = \lambda_{t,y} E_t[sdf_{t+1,m}(R_t^s - 1) r_{t+1}^{m})] = \lambda_{t,Y}(R_t^s - 1) E_t[sdf_{t+1,m} r_{t+1}^{m}] \tag{B2}$$

since R_t^s is known at the end of period *t*.

The Kuhn-Tucker conditions associated with $\mu_{t,y}$ are as follows:

$$\left\{ \mu_{t,y} \geq 0 \quad \text{and} \quad \mu_{t,y}(m_{t,y} - \delta) = 0 \right\} \tag{B3}$$

The second condition in (B3) is the complementary slackness condition. It implies that the cash constraint will be strictly binding iff $\mu_{t,y} > 0$ for all *t*.

Dividing (B2) by $1 = E_t[sdf_{t+1,m} r_{t+1}^{s}] = R_t^s E_t[sdf_{t+1,m} r_{t+1}^{m}]$, it follows that $\mu_{t,y} = \lambda_{t,y}(R_t^s - 1) / R_t^s$.

Since $\lambda_{t,y} = \partial U_t / \partial c_{t,y} > 0$, it follows that $\mu_{t,y} > 0$ iff $R_t^s > 1$ for all *t*.

An analogous argument applies to the cash constraint on the middle-aged using Equations (A6) and (A9). Therefore, it also holds with equality when $R_t^s > 1$. **Q.E.D.**

Section 2 – Numerical results and robustness analysis

2.A – Results for the baseline model when productivity risk is absent

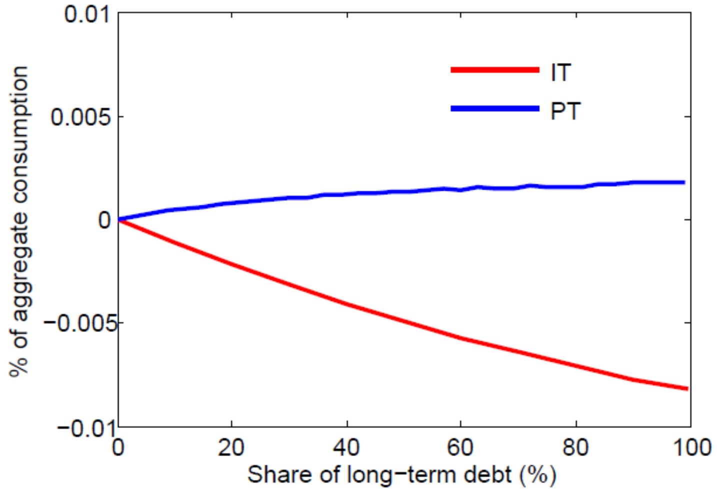

Fig A1 – Social welfare gain relative to the case of zero long-term debt (no productivity risk)

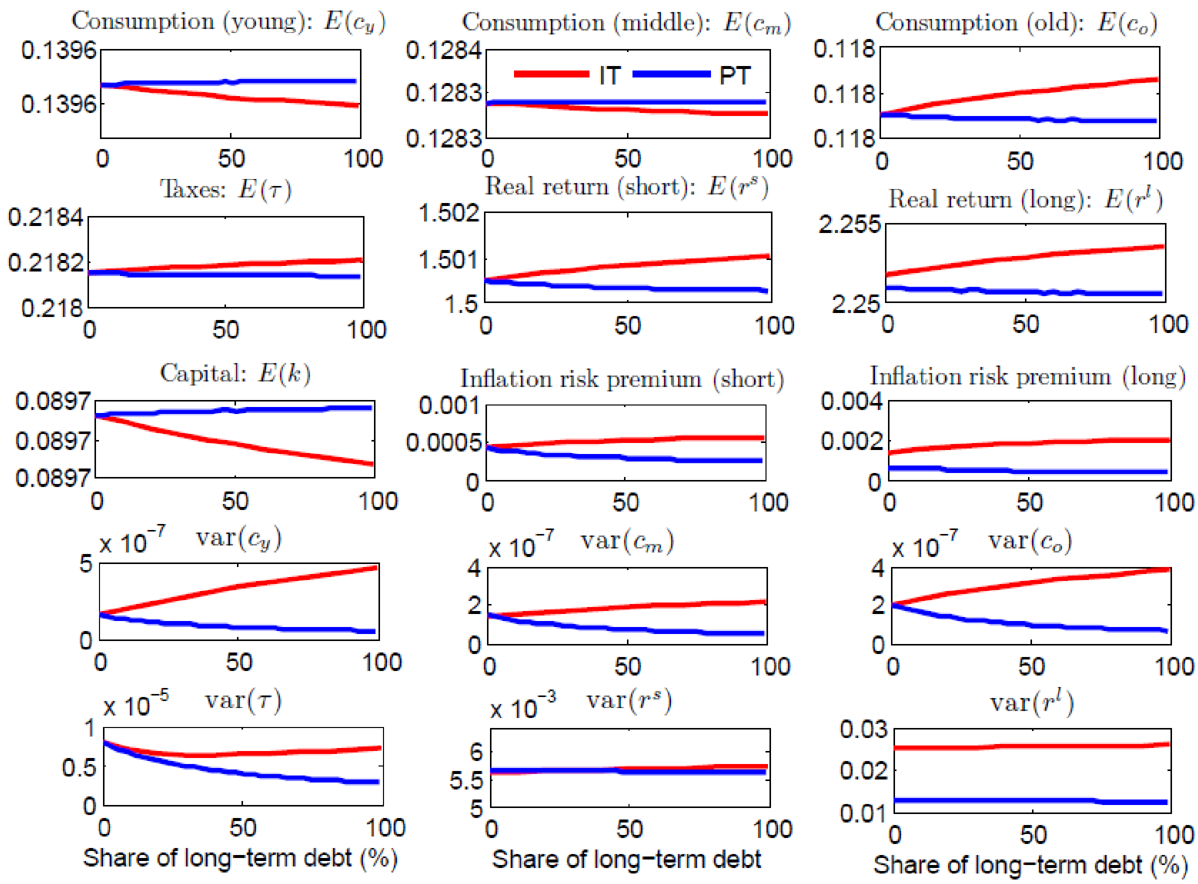

Fig A2 – Share of long-term debt and moments of real variables (no productivity risk)

2.B Utility of the initial old and middle-aged

In the baseline model, social welfare ignores the utility of the initial old and initial middle-aged. Therefore, as robustness check, social welfare was amended to include the utility of the initial old and the initial middle-aged as a robustness check. The amended social welfare function (SWF) is[34]

$$
\begin{aligned}
SW &= (1-\omega)E\left[\sum_{t=0}^{\infty}\omega^t U_t\right] + (1-\omega)[U_{t,m} + U_{t,o}] \\
&= E[U_t] + (1-\omega)[U_{t,m} + U_{t,o}]
\end{aligned}
\tag{B1}
$$

where $U_{t,m} \equiv \frac{c_{t,m}^{1-\gamma}}{1-\gamma}$ is utility of the initial middle-aged and $U_{t,o} \equiv \frac{c_{t,o}^{1-\gamma}}{1-\gamma}$ is utility of the initial old.

It is clear that the second line of (B1) converges on the original welfare function as $\omega \to 1$. Therefore, to determine if the results are affected, the model was simulated with SWF (B1) with $\omega = \epsilon$, where ϵ is a positive number close to zero. The results are reported in Section 7.1.

2.C Parameter sensitivity analysis

The 'high' and 'low' values of parameters used in sensitivity analysis are listed in Tables C1 and C2. The numerical results are too numerous to document here but are available from the author on request.

Table C1 – High calibrated values

α	Capital share	0.265	g^*	Government spending per period	0.095
β	Private discount factor	0.64	Π^*	Inflation target	NA
γ	Risk aversion	3.5	ρ_A	TFP persistence	0.5
δ	Real money holdings (m_y, m_m)	0.025	σ_e, σ_M	Std(e_t), Std ($\varepsilon_{M,t}$)	0.075
L_y	Labour supply (young)	0.60	L_m	Labour supply (middle-age)	0.40

Table C2 – Low calibrated values

α	Capital share	0.215	g^*	Government spending per period	0.075
β	Private discount factor	0.44	Π^*	Inflation target	NA
γ	Risk aversion	1.5	ρ_A	TFP persistence	NA
δ	Real money holdings (m_y, m_m)	0.005	σ_e, σ_M	Std(e_t), Std ($\varepsilon_{M,t}$)	0.025
L_y	Labour supply (young)	0.50	L_m	Labour supply (middle-age)	0.50

[34] Because the initial values in each simulation vary (due to a 'burn in' period), the utility of the initial old and initial middle-aged was computed using the average value across simulations.